

WORLD DYNAMICS: MEASUREMENT WITHOUT DATA^{1,2}

I. INTRODUCTION

IN 1798 T. R. Malthus published his influential *Essay on Population*, in which he argued that, when unchecked, population tends to increase faster than the food supply.³ He argued that if the eventual checks on population were the “positive” restraints of misery, pestilence, and famine, the population would be in equilibrium at the biological subsistence level. If the checks were the “moral restraint” of late marriage and sexual continence, the miseries of mankind might be alleviated. Although the Malthusian model may have some applicability to countries living on the border of subsistence,⁴ it is generally thought that Malthus could not foresee the tremendous technological advances of the industrial revolution as well as the tendency of higher standards of living to lower population growth.⁵

With much fanfare and alarum, Malthusian theories have recently been revived by a group of engineers and scientists. These contributions—of which the principal work is *World Dynamics* by M.I.T. Professor of Engineering Jay W. Forrester—have been described in an A.A.A.S. Bulletin as “a powerful new approach to understanding the dynamics of complex social systems”;⁶ by a prominent British scientist as “a method of forecasting

¹ The principal work reviewed here is Jay W. Forrester, *World Dynamics*, Cambridge, Mass., Wright-Allen Press, Inc., 1971. It was originally hoped that an analysis of its sequel, Donella H. Meadows *et al.*, *The Limits to Growth*, New York, Universe Books, 1972, could also be included. This was not possible because the simulation model was not published in the book. This revised version supersedes a preliminary paper (CF-20510) with the same title, May 1971. It has been revised after taking into account several comments of readers of the original paper. In particular, it has benefited from Jay W. Forrester's, “Response to a Paper on *World Dynamics* by Nordhaus,” mimeo D-1736-3, February 26, 1973. I am grateful to Forrester for his permission to take account of his response in this revised version.

² This research was supported by the National Science Foundation and the Ford Foundation. The author has benefited from discussion with William Brainard, G. M. Heal, Tjalling Koopmans, Richard Nelson, and Herbert Scarf, and from criticism by an anonymous referee, and the Editors. In addition, I would like to note my great debt to the writings and teachings of Robert Solow. None of the above is in any way responsible for opinions or errors in the paper.

³ T. R. Malthus, *An Essay on the Principle of Population*, etc., 1798.

⁴ For an early analytical exposition, see Robert M. Solow, “A Contribution to the Theory of Economic Growth,” *The Quarterly Journal of Economics*, February 1956, pp. 65–94. A thorough balanced discussion of the role of population in economic development is contained in W. Arthur Lewis, *The Theory of Economic Growth*, London: George Allen and Unwin Ltd., 1955, Ch. VI. For a discussion of the “low level trap” due to Malthusian population pressures, see H. Leibenstein, *A Theory of Economic-Demographic Development*, Princeton University Press, 1954; Richard R. Nelson, “A Theory of the Low-Level Equilibrium Trap in Underdeveloped Economies,” *American Economic Review*, December 1956.

⁵ See Harvey Leibenstein, *op. cit.*

⁶ Bulletin, *The American Academy of Arts and Science*, Vol. XXV, No. 7, p. 9.

the future which I consider as of the greatest importance and promise";¹ and by a prominent journalist as ". . . likely to be one of the most important documents of our age."²

In the spirit of Malthus, *World Dynamics* predicts an end to the economic progress that the West has experienced since the Industrial Revolution. The predictions are impressive to laymen and scientists alike because they appear to be derived from sophisticated models and extensive sensitivity analysis. The purpose of the present article is to re-examine the work in order to determine whether the extensive public acceptance is warranted. More precisely, the plan is (i) to examine the detailed specifications and assumptions from the viewpoint of economic theory and empirical findings (Sections III and IV); and (ii) to determine whether Forrester's results and predictions are sensitive to the specification of the model (Sections V and VI).

II. AN OVERVIEW OF THE APPROACH

The analytical technique used in *World Dynamics*—dubbed "systems dynamics" by the author—relies entirely on the use of computer simulations.³ The rather large model is composed of subsectors (*e.g.*, the population relations, the pollution rates). Each of the subsectors is composed of equations and graphs drawn from the author's knowledge. The behaviour of the system as a whole is traced out by a computer simulation.

The advantage of computer simulation is, of course, greater computational speed and precision than are available to the human mind. If the assumptions about functional forms and the data are accurate, simulations will lead to accurate predictions. On the other hand, without an accurate model there is no assurance that systems dynamics is better than mental models; the main result is a spurious and misleading precision.

The treatment of empirical relations in *World Dynamics* can be summarised as *measurement without data*. The model contains 43 variables connected by 22 non-linear (and several linear) relationships. *Not a single relationship or variable is drawn from actual data or empirical studies*. As Naylor and Finger state in an analysis of simulation in economic systems: "Analysts . . . have had little to say about how one goes about validating a simulation model or the data generated by such a model on a digital computer. . . . Simulation models based on purely hypothetical functional relationships and contrived data which have not been subjected to empirical validation are *void of meaning* . . . such a model contributes nothing to the understanding of the system being simulated."⁴ I would think that in such circumstances the author would be hesitant to proclaim his model the best available.

¹ Letter of Professor Dennis Gabor to *Science*, April 14, 1972.

² Anthony Lewis, *The New York Times*, January 14, 1972.

³ This is also true of his *Industrial Dynamics* and *Urban Dynamics*.

⁴ Thomas H. Naylor and J. M. Finger, "Validation" in Thomas H. Naylor (ed.), *Computer Simulation Experiments with Models of Economic Systems*, New York: Wiley, 1971, Ch. 5.

We can describe the *important assumptions* quite briefly. The important variables are shown in Table I, along with the conventional terminology and Forrester's nomenclature. In the discussion below, I will use standard economic terminology rather than Forrester's vague and often confusing appellations. The structural relationships in the model are as follows:

Population Growth is determined by *per capita* consumption, population density, food supply, and pollution. At poverty levels, population decreases rapidly; as affluence approaches, population grows very rapidly.

TABLE I
Important Variables in World Dynamics

Usual Names.	Name in <i>World Dynamics</i> .	Our Symbol.
<i>Stock Variables.</i>		
1. Reproducible capital	Capital-investment	K
2. Effective capital units <i>per capita</i>	Effective-capital-investment ratio	x
3. Population	Population	P
4. Land	Land	L
5. Exhaustible natural resources	Natural resources	R
6. Stock of pollution	Pollution	Pol
<i>Flow Variables.</i>		
7. Non-food production <i>per capita</i>	(No concept in <i>WD</i>)	y
8. Non-food consumption <i>per capita</i>	Material standard of living	c
9. Gross investment <i>per capita</i>	Capital-investment generation divided by population	s
10. Food production <i>per capita</i>	Food ratio	f

Note: Some of Forrester's variables can be interpreted with little difficulty. As is explained in the text, most of the flow variables are not defined in accord with usual economic conventions.

Here and below we follow the convention that upper case letters are totals, while lower case letters refer to *per capita* magnitudes.

Pollution is generated by economic activity and is absorbed only by natural processes. Emissions are proportional to population and are an increasing function of the capital-labour ratio. There is no way in which resources can be devoted to either reducing emissions or cleaning up the mess.

There are two factors determining *per capita output*: the capital-labour ratio and total resources remaining. There is no technological progress, no new discovery of resources, no way of inventing substitute materials, no price system to induce the system to substitute plentiful resources for scarce resources.

Per capita food production is not terribly important in the results. It is an inverse function of pollution and population. A slight increase in food production can be obtained by capital invested in agriculture.

All of these variables (and a few more) are linked in the simulation programme using 22 non-linear relationships. Most of the relationships are plausible, a few are not, but not a single relationship is tested empirically.

What are his *conclusions*? Forrester writes (pp. 11–13):

1. Industrialization may be a more fundamental disturbing force in world ecology than is population. . . .
3. We may now be living in a “ golden age ” when, in spite of a widely acknowledged feeling of malaise, the quality of life is, on the average, higher than ever before in history and higher now than the future offers.
4. Exhortations and programs directed at population control may be inherently self-defeating. If population control begins to result, as hoped, in higher *per capita* food supply and material standard of living, these very improvements may relax the pressures and generate forces to trigger a resurgence of population growth. . . .
6. There may be no realistic hope of the present underdeveloped countries reaching the standard of living demonstrated by the present industrialized nations. . . .
7. A society with a high level of industrialization may be nonsustainable. . . .
8. From the long view of a hundred years hence, the present efforts of underdeveloped countries to industrialize may be unwise.

Does Forrester see any way out of the dilemma? He suggests the following combination of policies, which lead to a “ global equilibrium ” in his model (p. 120):¹

Natural-resource-usage rate reduced by	75%
Pollution generation reduced by	50%
Gross investment reduced by	40%
Food production reduced by	20%
Birth rate reduced by	30%

He stresses many times that population control by itself is not enough: “ If growth is to be stopped . . . [d]oing this through direct population control will almost certainly fail ” (p. 113). In any case, the global equilibrium is approximately at today’s average world standard of living.

In describing his model Forrester states: “. . . I have greater confidence in this world system model than in others that I now have available. Therefore, this is the model I should use for recommending action ” (p. ix). Forrester’s theory of policy warrants a comment. The prescription just quoted argues for making policy decisions on the basis of the expected values of the model parameters, without considering either the quality of the estimates or the loss function associated with the outcomes. Forrester appears unaware that this “ certainty equivalence ” in policy recommendation depends on a very restrictive form of the loss function—that there be no uncertainty about the effects of policies *and* that the loss be a quadratic

¹ I have been unable to find a definition of “ equilibrium ” in *World Dynamics*: given the general tone, I assume equilibrium is one where all variables are constant.

function of the deviation from target.¹ Among other things, such a loss function implies that society's preference satiates at a finite level, and further that Forrester is not terribly averse to extinction of the human race.^{2,3}

There have been complaints from some reviewers that the level of aggregation in *World Dynamics* is too heroic. How can we expect to learn anything from so cosmic a model, aggregating all the world's people from Fiji to Santa Barbara? This criticism is puristic, for some aggregation is always necessary. The important question is, where can aggregation be performed without great cost? There is a considerable statistical literature on aggregation, all of which is ignored in *World Dynamics*. The main result of aggregation theory is that aggregation is generally possible only when the underlying micro relations are linear.⁴ In fact, few of Forrester's relations are linear, so the macro relations would generally depend on the distribution of the independent micro variables. This problem is particularly serious for population growth if we accept theories of population like that of the "demographic transition"—theories in which the functions are not only non-linear but have sign reversals.

III. THE SPECIFIC ASSUMPTIONS

We first evaluate the specific assumptions in the subsectors of *World Dynamics*. There are three serious problems—those involving population, technology, and prices.

(a) *Population*. The first crucial assumption in the model concerns the dynamics of population behaviour. The basic relations are as follows:

$$\Delta P(t) \equiv [b(t) - m(t)]P(t-1) \quad (1)$$

$$b(t) = B(c, f, d, Pol) \quad (2)$$

$$m(t) = M(c, f, d, Pol) \quad (3)$$

where P , c , f , and Pol are defined in Table I. b and m are the crude birth and crude mortality rates, respectively. d is population density. Unlike the simplest textbook models of economic growth, population growth in *World Dynamics* is endogenous to the economic system. Population growth responds positively to *per capita* food and non-food consumption, and negatively to

¹ See H. Theil, *Economic Forecasts and Policy*, Amsterdam: North Holland, 1961.

Again, we are being generous in our interpretation, since a close reading might show that Forrester is recommending a maximum likelihood outcome. I know of *no* general theorems leading to this criterion.

² Actually, Forrester proposes a log-linear "quality of life" utility function. This function is a very old one, dating back to D. Bernoulli in 1738. The use of such a criterion function is definitely inconsistent with the certainty-equivalent policy.

³ G. Heal has made a similar point to that made in this paragraph in an unpublished manuscript on the depletion of exhaustible resources.

⁴ The classic reference is H. Theil, *Linear Aggregation of Economic Relations*, Amsterdam: North Holland, 1954.

density and pollution. We have plotted in Fig. 1 the assumed response of population to consumption (c , or "the material standard of living"). This shows the assumed birth and mortality rates, holding *per capita* food consumption, density, and pollution at their 1970 levels. We are not yet able to state whether a stationary population exists, but if it does we see that (with other variables at their 1970 levels) the equilibrium comes with *per capita* non-food consumption at 70% of its 1970 levels.

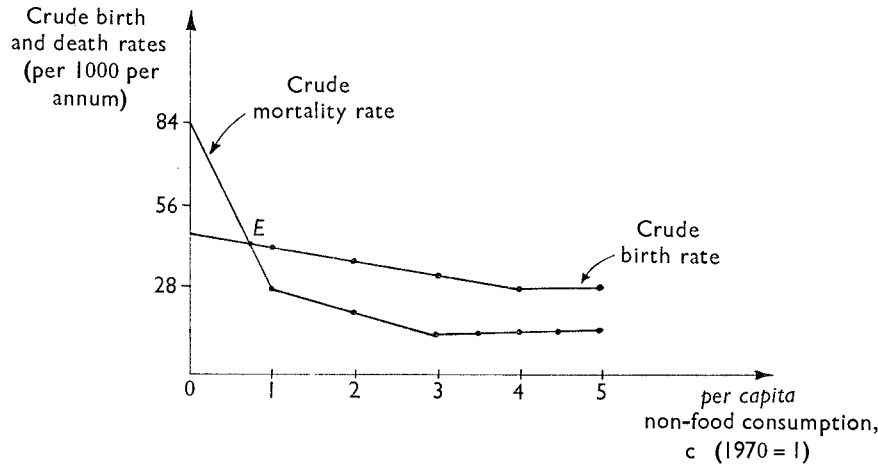


FIG. 1. Birth and Mortality Rates Assumed in World Dynamics.

Source: *World Dynamics*, Figures 3.1 and 3.3. It is assumed that *per capita* food consumption, population density, and pollution are held at their 1970 levels.

It is not correct to take the non-food consumption variable (c) by itself. The same general pattern is seen if one examines the predicted population growth in *World Dynamics* for all variables. Table II shows the Forrester prediction for birth and mortality development.

The general pattern of Forrester's model can be seen in Fig. 1 and Table II. The process of economic development generally leads to increasing levels of *per capita* food and non-food consumption. In *World Dynamics*, each of these lowers the mortality rate much more than the birth rate (see the difference between India and the United States in Table II). Offsetting these powerful effects Forrester has greater population density and greater pollution lowering net population growth, but the quantitative effects of these two forces is relatively small. Looking at the effect on growth in Table II, we see that, for the United Kingdom and the United States, Forrester predicts an increase in population growth over time while a decrease has actually taken place. Even more striking is the prediction for India in the last line of Table II. Forrester's model predicts a decline of population of 54% annually! The general pattern is clear: his assumptions

imply that affluent countries grow fast and poor countries decline, while exactly the opposite is seen in the data.

Are the assumptions in *World Dynamics* valid? The Malthusian model of population growth has generally been rejected by demographers and economists as inadequate for a general explanation of the behaviour of human

TABLE II
Illustration of Predicted and Actual Population Growth Rates and Determining Variables, Selected Countries and Years

Country.	Determining variables (World = 1 for 1970).				Birth, mortality, and growth (per 1000).					
					Forrester prediction.			Actual.		
	<i>c</i>	<i>d</i>	<i>f</i>	<i>Pol</i>	<i>b</i>	<i>m</i>	<i>b-m</i>	<i>b</i>	<i>m</i>	<i>b-m</i>
U.S.										
1900	2.11	0.30	2.13	0	57	8	49	32	17	15
1970	6.35	0.81	8.60	1	57	7	50	18	12	6
England and Wales										
1900	3.37	7.96	2.81	0	29	20	9	28	17	11
1970	5.41	12.00	4.53	1	31	21	10	16	12	4
India										
1970	0.25	6.22	0.27	0	70	610	-540	43	17	26

Note: Consumption and food *per capita* are deflated expenditures *per capita*. *d* is population density, while *Pol* is arbitrarily designated. For these estimates we have taken the estimate for World G.N.P. from *Limits to Growth* and assumed that world consumption is 0.75 times world G.N.P. Sources are the *U.N. Demographic Yearbook*, *U.S. Historical Statistics*, *U.K. Abstract of Historical Statistics*. The columns labelled "Forrester prediction" are calculated from the indices in the first four columns and from the relevant Figure in *World Dynamics*.

populations. It is sometimes thought, on the other hand, that it may be useful for explaining the behaviour of human populations living near subsistence.¹ The recent empirical evidence is very clear: on both a cross-sectional and time series basis, net population growth declines with increasing affluence. Fig. 2 shows a cross-sectional relationship between *per capita* G.N.P. and population growth, while Fig. 3 shows the time series for the United States, where *per capita* consumption rises over time.² The solid graph in Fig. 2 represents Forrester's assumed response of population to rising *per capita* non-food consumption when population density, pollution, and *per capita* food consumption is held constant.

¹ See Alfred Sauvy, *Théorie Générale de la Population*, 2nd edn., 2 vols., Paris: Presses Universitaires de France, 1956-1959; Bernard Okun, *Trends in Birth Rates in the United States Since 1870*, Johns Hopkins Press, 1958; Universities-National Bureau Committee for Economic Research, *Demographic and Economic Change in Developed Countries*, Princeton University Press, 1960; J. J. Spengler and O. D. Duncan, *Demographic Analysis*, Glencoe, Ill.: The Free Press, 1956.

² U.S. time series on population has been used because it is more familiar to the author. The same time path is found for other Western countries.

This neo-Malthusian theory of population behaviour is clearly unacceptable for the behaviour of countries which have reached a level of consumption on a par with Western countries. It is, however, an open question whether the behaviour of countries at a lower level of development looks more like the Malthus-Forrester assumption or the actual behaviour shown in

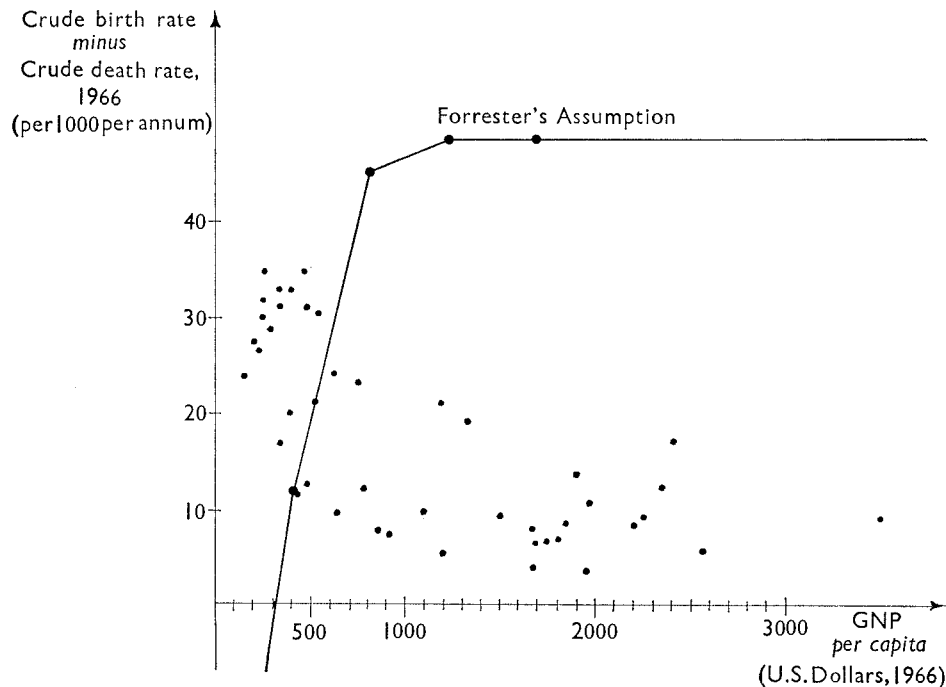


FIG. 2. Assumed and Cross Section Population Growth, 1966.

The line marked "Forrester's assumption" assumes that food and consumption rise in proportion with *per capita* G.N.P.; and that crowding and pollution are at 1970 levels.

Each of the unconnected dots represents one country: the sources of the data are the U.N. *Statistical Yearbook* and U.N. *National Accounts Statistics*, Vol. II.

Figs. 2 and 3. There is now a considerable body of knowledge relating to the "demographic transition" which suggests that the rate of population growth first rises then falls as a function of the level of development.¹ If the theory of the demographic transition is correct, then Forrester's assumptions are a serious mis-specification. Not only do they lead to incorrect results regarding the path of population growth, but they also point to very misleading implications for development policy.

(b) *Production*. A second area of World Dynamics which forms a crucial block of assumptions concerns the equations for capital, consumption, and accumulation. We will concentrate only on the non-farm sector. In this

¹ H. J. Habakkuk, "English Population in the Eighteenth Century," *Economic History Review*, 2nd, ser., vol. 6, 1953; Ansley Coale, "The Voluntary Control of Human Fertility," *Proceedings of the American Philosophical Society*, June 1967.

sector there is explicit treatment of capital accumulation and resource exhaustion. There is also an implicit notion of consumption. The concept of output, and especially the notion of a production function, are missing. The effective capital-labour ratio (x) is given by:^{1,2}

$$x_t = ARK_t/P_t \quad (4)$$

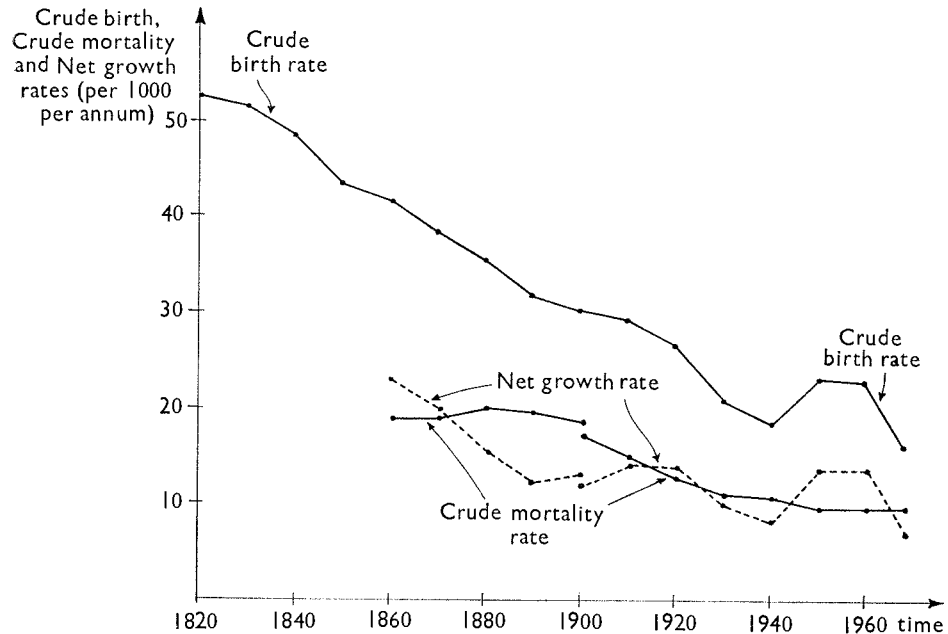


FIG. 3. Birth and Death Rates, U.S., 1850-1970.

Source: Data are from U.S. Department of Commerce, *Historical Statistics* and updated from the *Statistical Abstract*. The data is for the white population of U.S.A., except that the death rate 1860-1900 covers only Massachusetts. The usual precautions about use of gross birth and death rates are, of course, applicable to these data.

Recall that R is the stock of natural resources, K is capital stock, and P is population, while A is an arbitrary constant. *Per capita* non-farm consumption (c) is proportional to the effective capital-labour ratio (Section 3.4):

$$c_t = ax_t \quad (5)$$

Capital accumulation is (Sections 3.24 to 3.27):

$$\Delta K_t = s(c_t)P_t - \delta K_t \quad (6)$$

¹ We also omit from the exposition the share of capital in agriculture. We have treated this function as linear in R , whereas Forrester's form (Fig. 3.2, p. 37) is slightly non-linear. Equation (4) is from *World Dynamics*, Sections 3.5 and 3.6.

² The variable x is the "effective-capital-investment ratio." *World Dynamics* states that " x consists of those capital units that contribute directly to improving the standard of living. The number of effective units per person is arrived at by applying to total capital units a discount that neglects any shortage in world-wide natural resources" (p. 36).

where $s(c_t)$ is the gross investment rate per person and δ is the exponential rate of depreciation of capital. The $s(c_t)$ function is shown in Fig. 4. $s(c_t)$ rises from a low of 0.005 at zero income and saturates at 0.15. Finally natural resources are exhausted as indicated in equation (7):

$$R_t = R_{t-1} - \theta(c_t)P_t \quad (7)$$

θ runs from 0 to 4 as c runs from 0 to 10.

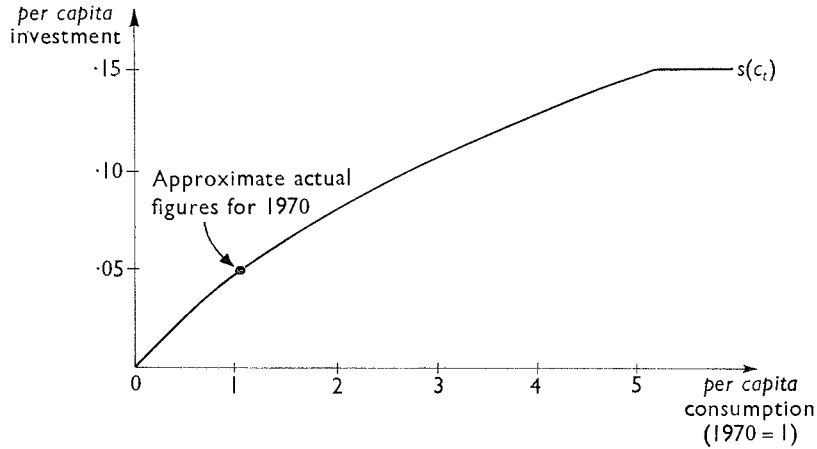


FIG. 4. *Per Capita Investment and per capita consumption.*

The figure shows the level of investment per head assumed by Forrester to correspond with each level of consumption per head. Note the saturation of investment per head at three times its 1970 level. Source: Forrester, Figure 3.12.

Considering the non-farm sector only, we can derive non-farm *per capita* gross output (y) from (4) and (6) as follows (using capital as a numéraire):

$$y = c + s = ax + s(c)$$

or

$$y = ax + s[c(x)] = f(x) \quad (8)$$

where $c(x) = aARK/P$.

At first blush the production and accumulation relations may appear plausible. On further reflection they can be shown to be inconsistent with the production functions and accumulation relations that economists have been studying, analysing, and estimating for fifty years.¹ First, it appears that the fraction of output *invested* (as expressed in the s function) influences the *level* of output. More specifically, a unit increase in investment increases output by exactly the same amount. Forrester has introduced a widow's

¹ For comprehensive surveys, see A. A. Walters, "Production and Cost Functions: An Economic Survey," *Econometrica*, January/April, 1963; Edwin Mansfield, *Industrial Research and Technological Innovation*, Norton, 1968; and Charles Kennedy and A. P. Thirlwell, "Surveys in Applied Economics: Technical Progress," *ECONOMIC JOURNAL*, March 1972.

cruse in the production relations, so there is no longer any trade-off between factories and butter.

Second, note that the *only* input into the level of *per capita* output is the effective capital-labour ratio, x . Recall from (4) that:

$$x = ARK/P \quad (4)$$

Ignoring for the moment the second term in (8), output is a linear function of x .¹ Considering (8) as a normal production function, and using (4), we discover dramatic increasing returns to the scale of the economy: if we double both the number of blast furnaces and the number of ore fields the output of pig iron *quadruples*. Economists, and presumably engineers as well, would guess that output would more or less double [as in $(R_t K_t)^{1/2}$] rather than quadruple. The reason this is crucial is that Forrester is running the system in reverse. A halving of both capital and resources divides output by four. This pessimistic assumption makes the system grind to a halt very fast.

The bizarre results are a reflection of the general problem that *World Dynamics* does not have a clear specification of a production function. Note that all inputs into the implicit production function are *stocks* rather than flows or services. Forrester argues that resource *stock* enters because a decreasing stock of natural resources has a negative effect on efficiency: "if there are no natural resources, capital investment will be ineffective" (*World Dynamics*, p. 37). Forrester has collapsed two considerations into one variable. (i) It is a classical economic proposition that extractive industries are increasing cost industries: for given technology this simply means that marginal cost is a decreasing function of "resources remaining." (ii) The second point, however, is that the appropriate concept of resource inputs in a production function is the flow of materials, not the stock of resources remaining. Thus a satisfactory specification would involve two production functions—one for the extractive industry and one for the rest of the non-farm economy. Collapsing considerations (i) and (ii) is possible only when there is no substitution possible between resource input and other inputs.

One very misleading implication of Forrester's specification is that if the rate of usage of natural resources slows down, *output goes up rather than down* (compare Figs. 4.1 and 4.5 in *World Dynamics*). The reason is clear: as we slow down depletion, natural resources remaining fall less rapidly and output in equation (4) goes up. This is not a terribly important detail, but it shows how careless specification of functional forms leads to absurd results. A second questionable assumption concerning resources is that there is a fixed stock of exhaustible non-reproducible resources: according to Forrester, the supply is 400 times the current annual consumption. This specification is very primitive and through improper aggregation begs the crucial questions

¹ Including the second term would reinforce the argument.

about resource availability. In fact, there are many kinds of "natural resources." If we go to the fundamental building blocks—the elements—the degree of scarcity varies tremendously. For some elements, the ratio of known reserves to consumption is very small; gold, mercury, and silver have reserve-annual consumption (R/C) ratios of slightly greater than 10. For many others, the R/C ratio is virtually infinite. Thus oxygen, nitrogen, carbon, and chlorine have R/C in the order of 100 million. Forrester is implicitly assuming that there is no possibility of substituting the abundant resources for the scarce. There have been many counter-examples to Forrester's assumption: iron, aluminium, and communication satellites replace copper; chlorine replaces iodine; the xerography process replaces use of tin and lead in printing.¹ The impossibility of substitution is also seen in equation (7), where the amount of resource consumption is bound to population and consumption by an iron law of resource use. There is no way of substituting plentiful for scarce resources, nor is there any way of substituting abundant capital and labour for dwindling natural resources.² A more conventional economic view would be that more resource-intensive processes and products will become more expensive, and that the substitution of less resource-intensive processes and products will lead to conservation of scarce resources. Whilst it cannot generally be guaranteed that sufficient possibilities for substitution exist, there can be no general theoretical case for the highly pessimistic assumptions about substitution which are made in *World Dynamics* and Forrester has not provided adequate evidence to justify his assumption.

We return to an analysis of Forrester's model. Given the iron law of resource use, it is obvious that the system will gradually grind to a halt. Even though resources get more and more scarce and produced goods get more plentiful, Forrester's economy continues to gobble up resources. Perhaps extinction is the just fate for such insentient men. But man has in fact been far more adaptive. Coal replaced the vanishing charcoal in eighteenth-century England, nuclear fuels are replacing disappearing fossil fuels in the twentieth. It is possible that the historical ability to find substitutes for scarce resources will vanish, but this would make the future very different from the past.

A final questionable, but probably not crucial, assumption concerns the accumulation equation (6). This equation shows that the ratio of gross investment to non-farm output falls very sharply as *per capita* consumption rises. It is not possible to derive the exact form of the relation because both c and x are in index form. If Forrester relied on information used by his

¹ The data and some examples in this paragraph are drawn from H. E. Goeller, *A Mineral Resources Primer: Part I*, mimeo, 1972.

² There is yet another objection to the Forrester model of resources. Strictly speaking resource utilisation does not consume resources but rather it quite literally downgrades them. What were highly concentrated resources become widely dispersed. Whether secondary production (or recycling) is possible depends on the grade of the secondary material.

collaborators in *The Limits to Growth* (p. 112), then he would have set the present level of *per capita* G.N.P. at \$400. World consumption *per capita* is approximately 75% of that, \$300. This implies that $c = 1$ corresponds to \$300 *per capita*. He does not discuss what the constant a in equation (5) corresponds to. If we are to measure consumption using capital as numér-

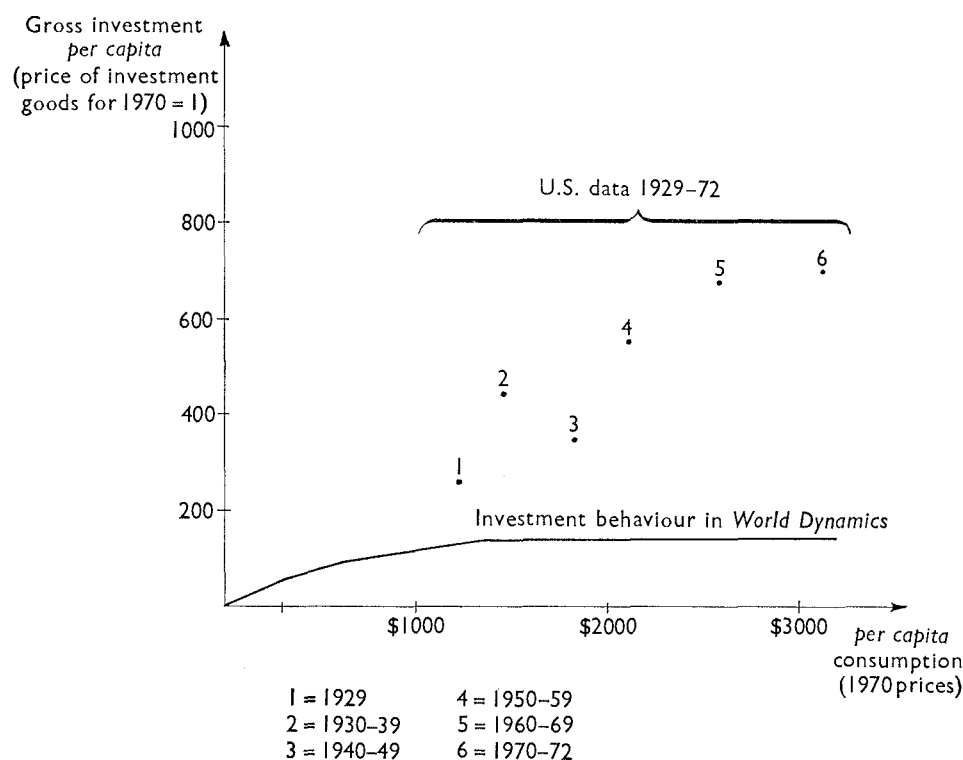


FIG. 5.

Solid line shows implicit investment behaviour in *World Dynamics*. Dots show actual investment behaviour for the U.S. for the period 1929-1972.

aire, a reasonable guess for a would be that consumption is perhaps one-third of the value of the capital stock in 1970, so that at $c = 1$, the existing capital stock *per capita* is \$900, and *per capita* investment is currently about \$45; and that investment *per capita* satiates at about \$135 *per capita* as industrialisation takes place.¹ In reality, *per capita* gross investment is about \$750 *per capita* in the United States.² In time series studies, it has generally been found that the ratio of savings to income is constant.³ Fig. 5 plots the level

¹ Forrester assumes *per capita* investment is 5% of 1970 *per capita* capital, say \$900. He further assumes that investment satiates at three times the 1970 level, e.g., $3 \times 0.05 \times \$900 = \135 *per capita*.

² See *Economic Report of the President*, 1973.

³ For example, see Simon Kuznets, *Capital in the American Economy*, Princeton University Press, 1961.

of *per capita* gross investment against consumption assumed by Forrester. For comparative purposes, a time series of the actual behaviour of the gross investment–consumption ratio for the United States is shown.

(c) *Allocation of resources.* Finally one notes that there is no explicit mechanism for allocating resources over time and between sectors. Economists usually introduce prices as an allocating mechanism. This is a crucial omission in Forrester's system, for prices are one obvious adaptive mechanism by which economic man does adjust to changes in relative scarcities such as those Forrester describes. *If* there is sufficient substitutability between producible and non-producible resources and *if* the price system is functioning adequately, the inevitable collapse predicted by Forrester will be avoided. (More on this in Section VI below.)

The assumption about prices fits into the same mould as that about population, technology, and resources: human society is a population of insentient beings, unwilling and unable to check reproductive urges; unable to invent computers or birth control devices or synthetic materials; without a price system to help ration scarce goods or motivate the discovery of new ones.

IV. A CLOSE LOOK AT THE SIMULATIONS

The next four sections focus on the overall properties of the model presented in *World Dynamics*. The present section is basically analytical, stripping the model to its essentials and examining its dynamic properties.

(a) *Equilibrium with no resource constraint.* It will be instructive to investigate first the population and production equations, for these are the heart of the model. We will later reintroduce the resource constraint. We can rewrite the population equation as

$$P(t) \equiv P(t-1)\{1 + H[c(t-1)]\} \quad (9)$$

where H is shown as the difference between birth and death rates in Fig. 1.¹

For output and consumption, we relax the natural resource constraint in equation (4) setting $AR_t = 1$; further we set $a = 1$ in equation (5); by substitution we obtain a difference equation in *per capita* consumption as:

$$\frac{\Delta c_t}{c_t} = \left\{ \frac{s(c_t)}{c_t} - \delta - H(c_t) \right\} \quad (10)$$

where the *per capita* investment function, $s(c_t)$, was shown above as Fig. 4. Equation (10) is what we call the *unconstrained basic model*.

We have shown equation (10) graphically in Fig. 6. For ease of exposition we have divided the function into two parts, $\{s(c)/c\}$ and $\{\delta + H(c)\}$,

¹ We have left out the three variables representing food, pollution, and density in Forrester's specification.

the parts having been constructed from the numerical values of $s(c)$ and $H(c)$ shown in Figs. 1 and 5. From equation (10), we know that *per capita* consumption is in equilibrium when the graphs of the two parts intersect. This represents equilibrium in the model when there are no limitations due to crowding, resources, or pollution. When there are no such con-

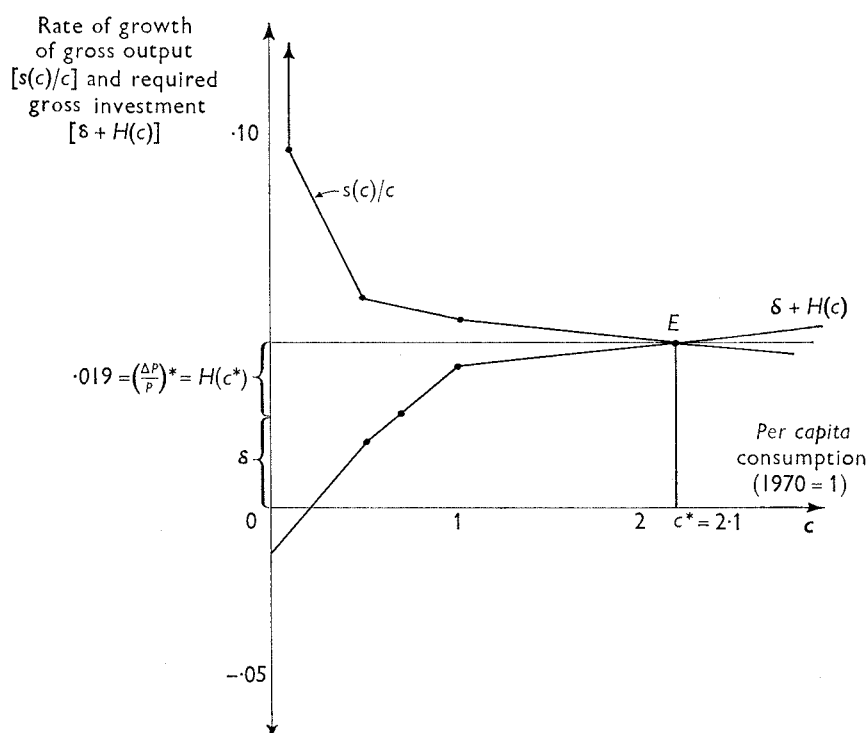


FIG. 6. Equilibrium in Forrester's Unconstrained Basic Model.

This shows the long-run equilibrium in the model when it is unconstrained by resources, land, or pollution. The basic relation has been derived in equation (10). The downward sloping line $[s(c)/c]$ shows the rate of growth of gross output; while the upward sloping curve $[\delta + H(c)]$ shows the rate of growth of required capital. At equilibrium point E , *per capita* consumption is constant; population and capital grow at 1.9% per annum.

straints, equilibrium occurs with c^* at 2.1 (that is, *per capita* consumption 210% of the 1970 levels), with no growth in *per capita* consumption, and with population growth at rate 1.9% annually. If an unconstrained equilibrium gives a sombre outcome, little imagination is needed to discern what happens with limitations on essential resources, Malthusian limits to food, population, and crowding.

It is also easily verified that c^* is a globally stable equilibrium: to the left of c^* in Fig. 6, $s(c)/c$ exceeds $\delta + H(c)$, so c is rising; to the right c is falling. This simple analysis is useful in showing the upper limit on the

world economy in Forrester's system. The one-equation model predicts that with no natural resource or other constraints, the system tends toward $c^* = 2.1$. In fact, this tendency shows up in Figure 4.10 of *World Dynamics* where natural resources and pollution are removed as constraints.¹

For later amplification we have constructed numerical approximations to $s(c)$ and $H(c)$:²

$$s(c) = 3 - \frac{6.44}{2.22 + c} \quad (11)$$

$$H(c) = 0.181 - \frac{0.765}{3.529 + c} \quad (12)$$

Setting up these as a two-equation difference equation model, we get the simulation shown below:

	Equilibrium: Per capita consumption (c^*).	Population growth $\left(\frac{\Delta P}{P}\right)^*$.
Fig. 6	2.1	%
Simulation	1.25	1.9 2.1

The results are basically the same as in Fig. 6 except that the interpolation of $H(c)$ is slightly inaccurate.

(b) *Equilibrium with finite essential resources.* The unconstrained model shows how the system behaves in unconstrained situations when assumptions concerning the investment and population behaviour are pessimistic. We next introduce the resource constraint in equation (7) and a finite supply of exhaustible resources.³

The full system can be simplified by assuming $s(c) = \bar{s}c$, which approximates Forrester's assumptions for the range $0 \leq c \leq 1$. Further, $\theta(c)$ is linear in the range $c \leq 1$, so $\theta(c) = \bar{\theta}c$ where $\bar{\theta}$ is a constant. We ignore

¹ The simulation in *World Dynamics* Figure 4.10 removes the constraint on resources and pollution but not *food*, and the equilibrium is almost precisely that shown in Fig. 6. It is strange that food does not appear to have any substantial effect on the system's equilibrium when crowding suppresses population growth.

² These were obtained by fitting hyperbolas to three points in the relevant figures in *World Dynamics*.

³ Forrester would lead us to believe that the human mind is incapable of analysing such systems, and that the techniques of "systems dynamics" are necessary. As in the unconstrained model above the reverse is probably the case. A little simple algebra will allow us to understand not only the system's behaviour, but also (unlike systems dynamics) we can get a general feeling for the crucial assumptions.

pollution and food. Writing (4) and (7) in continuous form and using the simplifications just mentioned, we obtain:

$$\frac{\dot{K}}{K} = A\bar{s}R - \delta \quad (6')$$

$$\frac{\dot{R}}{R} = -\bar{\theta}AK \quad (7')$$

where dots over variables represent time derivatives.¹

It can be shown that there is a single general solution to (6') and (7'). The solution involves a diminishing absolute stock of capital; resources tending to a positive limit; population eventually declining at the same rate as capital; and consumption stabilising at a low level.

To see this, consider a path with the asymptotic stock of resources, R^* , such that $0 \leq R^* < \delta/\bar{s}A$. This leads to capital decline at rate g , where $0 > g = \bar{s}R^*A - \delta \geq -\delta$ and to resource consumption at rate $\dot{R}/R = -\bar{\theta}K^*e^{-\theta t}$. Thus at time t , $R_t = R_0 \exp\left[\int_0^t (\dot{R}_v/R_v)dv\right] = R_0 \exp\left[-\bar{\theta}K^* \int_0^t e^{-\theta v} dv\right] = R_0 \exp[(-\bar{\theta}K^*/g)[1 - \exp(-gt)]]$. So $R \rightarrow R^* = R_0 \exp(-\bar{\theta}K^*/g)$ as $t \rightarrow \infty$. Using (6') we get the condition $g + \delta = A\bar{s}R_0 \exp[\bar{\theta}K^*/g]$, which defines the asymptotic path and growth rate. Given the initial conditions of $\bar{s} = 0.05$, $\delta = 0.025$, and $\bar{\theta} = A = 0.00131$, $R_0^* = 760$, $K_0^* = 3.6$ the system has a solution with $g = -3.3\%$ per annum. This should come with very low levels of consumption, perhaps $c = 0.2$ if Fig. 1 is consulted and with population declining at 3.3% annually.

To summarise the analytical results: An analysis of Forrester's model when unconstrained by food, pollution, and land shows that it has a stable equilibrium at a low level, with *per capita* world consumption approximately double the current level. When the limitation of scarce, exhaustible, and essential resources is added, the system has an asymptotic path with population declining toward extinction at about 3% annually and a consumption rate of about one-fifth of the current level.

¹ Equation (7') follows directly from (7) when we substitute from (1) and the linear form $\theta(c)P = \bar{\theta}cP = \bar{\theta}AR(K/P)P = \bar{\theta}ARK$. For equation (6'), we start with

$$\frac{\dot{K}}{K} = s(c) \frac{P}{K} - \delta$$

Since $s(c) = \bar{s}c$ and from (1),

$$\frac{\dot{K}}{K} = \bar{s}AR \frac{K}{P} \cdot \frac{P}{K} - \delta = \bar{s}AR - \delta$$

V. A MODEL OF A MODEL

Section IV introduced in equations (11) and (12) certain simplifications of the model. We next introduce pollution and the agricultural sector in order to obtain a simple version of the full model used in *World Dynamics*.

Pollution. Like capital, pollution is generated and absorbed. The generation of pollution is proportional to population and increases with the capital-labour ratio. Absorption is by natural forces alone (there can be no pollution abatement programmes) and is inversely related to the stock of pollutants. The simulated form, for which the only simplification is using a smooth function, is:

$$\Delta Pol_t = \alpha_0 P_{t-1} - \alpha_1 Pol_{t-1} \quad (13)$$

$$\alpha_0 = \min \left[0.625 \frac{K}{P}, 8 \right]$$

$$\alpha_1 = 1 / \left[0.5 + 0.25 \frac{Pol_t}{Pol_0} \right]$$

Food. *Per capita* food production (f) is an increasing function of capital and a decreasing function of population and pollution. As noted above, capital has much less effect on food production than does pollution. We therefore simplified the agricultural sector to the following interpolated schedule:

$$f_t = \left[1 - \frac{Pol_t}{60} \right] \frac{6}{P_t + 3.6} \quad (14)$$

Finally we must take account of Forrester's treatment of capital in agriculture. The following approximates roughly his assumption:¹

$$\sigma = 0.6 [\max(1 - 0.5f, 0.1)] \left[\min \left(0.7 + 0.3 \frac{c}{f}, 2 \right) \right] \quad (15)$$

where σ is the share of capital in agriculture. This enters the system through equation (6); which becomes

$$\Delta K_t = (1 - \sigma_t) s(c_t) P_t - \delta K_t \quad (6'')$$

Equations (1), (5), (6''), (7), (13), (14), and (15) are the system we will use to perform sensitivity analysis on the assumptions of *World Dynamics*, and these equations will be called the *simple model*.² This replaces the equation

¹ It is tiresome, but necessary, for us to repeat that there is not a shred of evidence adduced to support the assumptions in (13), (14), or (15).

² A computer printout of the simple model will be made available on request.

in 43 variables and 22 non-linear functions in Forrester's simulations; we refer to the model as published in *World Dynamics* as "Forrester's full model." Fig. 7 shows the values in our simple model and in Forrester's full model. As

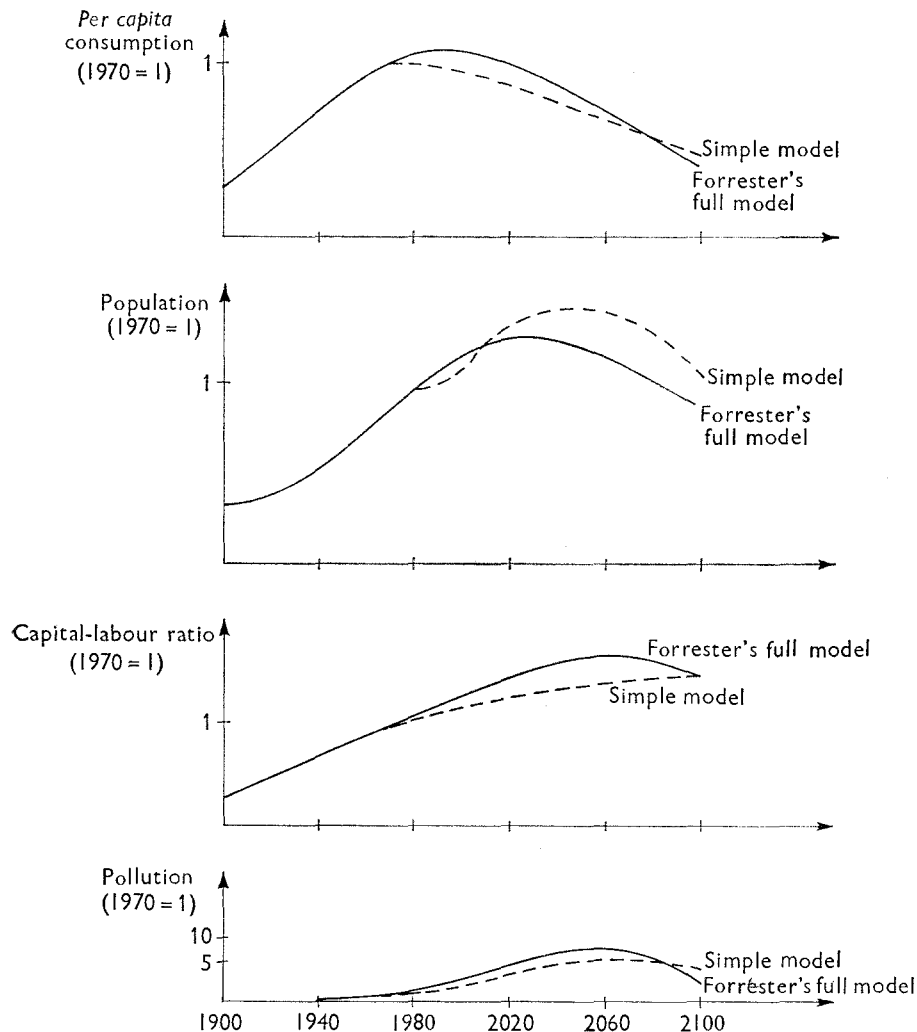


FIG. 7. Comparison of Simulations of Simple Model and Forrester's Full Model.

The simple model consists of equations (1), (5), (6''), (7), (13), (14), and (15) in the text. The results for Forrester's full model reproduce the basic behaviour in *World Dynamics* as described in Figures 4.1 and 4.2.

can be seen, the simple model tracks the larger model quite well. It should be cautioned that with one exception, the results below apply, strictly speaking, to the simple model rather than Forrester's full model.

The asymptotic behaviour of Forrester's full model is not discussed in Forrester. In the simple model outlined here, consumption stabilises at

$c^* = 0.36$, with population falling at 1.6% annually. This confirms the theoretical analysis of equations (6') and (7') in Section IV above.

Having satisfied ourselves that the simple model operates in a similar fashion to Forrester's full model, we now turn to some sensitivity analysis.

VI. SENSITIVITY ANALYSIS: ASSUMPTIONS

In the discussion of Section III, we outlined several objectionable features of Forrester's assumptions—assumptions which are both theoretically implausible and contrary to the available empirical studies. We shall correct these objectionable features and see if the behaviour of the simple model outlined above changes.

Assumption A. Population. We showed earlier that Forrester's assumptions about population are at variance with the observed cross-sectional and time series relationships between population growth and consumption.

There is great uncertainty about future population trends.¹ We have taken three different assumptions:

- A1. Population grows at 2% annually.
- A2. Zero population growth (ZPG).
- A3. Population declines at 2% annually.

Further, for assumptions B, C, and D, we will have four variants. Assumptions BO, CO, and DO use Forrester's population assumption; Bi, Ci, and Di use population assumption Ai; where $i = 1, 2$, and 3.

Assumption B. Investment-Savings. It is customary in growth models to assume that investment is approximately proportional to income, say around the 12% that has been observed in the United States. We will substitute this for Forrester's saturation assumption as assumption B.

Assumption C. Technological Change. It is assumed that the unit requirements of labour, capital, resources, and land in the productive process diminish as a result of technical progress. Although there are a large number of possibilities in this area, a figure which is reasonably consistent with past behaviour is a decline in input requirements per unit of output of 2.5% annually.

Assumption D. Substitutability of Inputs and Outputs. We have noted that Forrester assumes that there is no inducement for the economic system to substitute plentiful for scarce factors, or to find means of reducing lethal pollution. Assumption D allows limited substitutability between inputs and outputs of the economic system. Unfortunately, we cannot say that it is

¹ The heroic aggregation in *World Dynamics* is most dangerous at this point: as noted, aggregation errors are most likely to be serious when the individual relations are non-linear. If we accept the plausibility of the theory of the demographic transition, it is very difficult to predict the course of world population without knowing the distribution of world income.

well grounded in empirical findings, although some aspects of this assumption have been tested.¹ There are four inputs into the productive system and three outputs. The inputs are the services of land, labour, and capital, and the consumption of natural resources.² The outputs are two "goods"—food and non-food output—and one "bad", gross emissions of pollutants. We can write the system as

$$\phi(\pi, F, X; P, K, -\Delta R, L) = 0 \quad (16)$$

where π = gross emission, F = aggregate food output, $X = C + G$ = gross non-farm output, P = population = labour inputs, K = capital services = capital stock, $-\Delta R$ = resource consumption, and L = land. There are a large number of possible function forms for (16). Forrester assumes, as noted above, that there is no substitutability between pollution, food, resource depletion, and output. Since there is engineering and economic evidence of the availability of ample alternative processes of production, we assume for purposes of simulation that there is some substitutability between inputs and outputs. First, we follow a considerable body of economic investigation in assuming that inputs are combined in a log-linear fashion (this being the well-known Cobb–Douglas production function).³ For simplicity, let N be composite inputs, where

$$N = P^{\beta_1} K^{\beta_2} (-\Delta R)^{\beta_3} L^{\beta_4} \quad (17)$$

We further specify that $\sum_{i=1}^4 \beta_i = 1$, implying constant returns to scale. Let us stress that we do not think (17) is necessarily the proper specification: it may imply too much substitutability, or not enough. It is roughly in accord with the macroeconomic facts of the past half century.⁴

On the output side, there is less evidence to rely on. In assessing the cost of clean technologies, it is generally thought that, with a given *process*, gross emissions and output are joint products which increase proportionally with the scale of operation. In Fig. 8, with inputs of \bar{N} , the process produces FD units of output and ED units of emissions.

Forrester assumes that there is a single process available, but in general there are alternative processes. Considering substitution available with fixed inputs \bar{N} , the curves CDB and ODB are alternative possibilities. If

¹ See W. D. Nordhaus and J. Tobin, "Is Growth Obsolete?" in *Economic Growth*, Colloquium V, NBER, New York, 1972, esp. Appendix B; H. Barnett and C. Morse, *Scarcity and Growth*, Resources for the Future, 1963.

² This corrects one of Forrester's errors which puts the *stock* of natural resources into the production function.

³ See Marc Nerlove, *Estimation and Identification of Cobb–Douglas Production Functions*, Chicago: Rand-McNally, 1965; also Walters, *op. cit.*

⁴ See J. W. Kendrick and R. Sato, "Factor Prices, Productivity, and Economic Growth," *American Economic Review*, December 1963; J. Kendrick, *op. cit.*; and E. Denison, *The Sources of Economic Growth*, Committee for Economic Development, 1962; Nordhaus and Tobin, *op. cit.*

ODB is the curve representing possibilities for substitution, then there is no completely clean technology; while if CDB is the true curve, OC represents the output available with a clean technology.

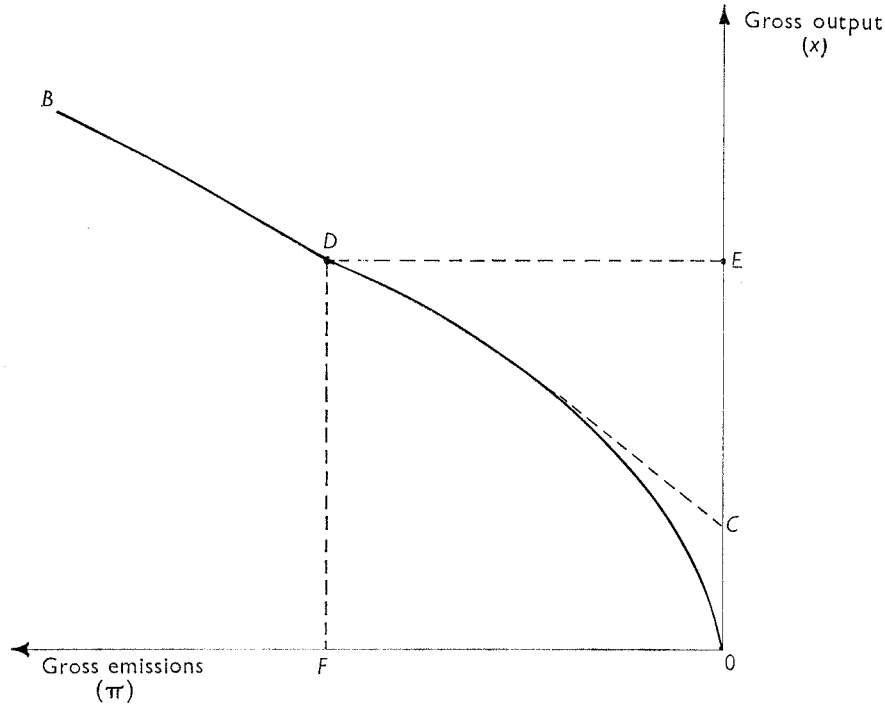


FIG. 8. Trade-off between Gross Emission and Gross Output.

Forrester assumes a fixed proportions relation between Gross Emissions and Gross Output, so that with given inputs only one combination is possible—*e.g.*, as shown at D . In general, substitution is possible, so that alternative combinations are possible, as along BDC or BDO . If the substitution curve hits the vertical axis at positive levels (say OC), this represents a completely clean technology.

A simple algebraic form for curve CDB is

$$X/N = c(\pi/N + a)^b$$

or

$$X = cN^{1-b}(\pi + aN)^b \quad (18)$$

where $a \geq 0$ and ca^b is the maximum clean output per unit input; we set b such that $0 < b < 1$; and c is an arbitrary constant. The condition that $b < 1$ simply reflects the well-known phenomenon of diminishing returns: the marginal cost of reducing emissions per unit output increases as the desired scale of reduction rises. Finally, we assume food and non-food output are perfect substitutes in production.¹

¹ We referred above to the importance of prices in the allocation of scarce resources (see p. 1169 above). The specification in (16) hides a great deal of undercover behaviour. In particular, it is

Introducing technical progress at rate h and collecting terms, we have if $a = 0$

$$F + X = ce^{ht}[P^{\beta_1}K^{\beta_2}(-\Delta R)^{\beta_3}L^{\beta_4}]^{1-b}\pi^b \quad (19)$$

If $a > 0$, we have to expand the more complicated equation given by (17) and (18).

We stress that this formulation has not been empirically tested. From scattered evidence on the current cost of introducing cleaner technologies in automobiles and electricity generation, a value of b of 0.1 seems reasonable. To be conservative, we assume there is no clean technology, so $a = 0$.

Finally, we must make assumptions about the relative amounts of inputs and outputs and the following arbitrary assumptions seem reasonable: land is fixed; labour and capital are determined according to assumptions A and B ; for resources we assume that a fixed fraction of remaining resources, ρ , is used every year.¹ As an upper limit, we assume that 5% of resources are used annually, so $\rho = 0.05$. These four assumptions determine the inputs available at any time. We assume that the amount of pollution is set by law at some point below the upper limit that the ecosystem will permit, at $\bar{\pi}$.² It is usual to assume that under competitive pricing the share of the factor in national income will be equal to the coefficients in equation (16).³ We can thus find approximately that $\beta_1 = 0.7$, $\beta_2 = 0.2$, $\beta_3 = 0.05$, and $\beta_4 = 0.05$. These values form the basis for our simulations for assumption D.

VII. SENSITIVITY ANALYSIS: RESULTS

We cannot report all the results of the sensitivity analysis, nor must we take any of these too seriously. Recall that the purpose of this exercise is simply to determine if the results of the Forrester model are sensitive to what we think will generally be acknowledged as errors in model specification.

The main result is what happens to the path of *per capita* consumption under different assumptions. This is shown in Figs. 9 to 12. Each figure

generally assumed that all inputs and outputs are allocated by competitively determined relative prices. This ensures that firms find the least-cost processes for producing the bundle of outputs. If the price system malfunctions—as is currently the case for free but scarce public environmental resources—then perverse outcomes are possible.

¹ This crude assumption is made mainly for the purposes of convenience. It can be shown that it is the optimal policy in certain circumstances.

² Then we know that $dPol/dt = \pi - A$, where A is the absorption in the system. The main determinants of absorption are the level of pollution and the scale of clean-up activities. Omitting clean-up, we can write $A = A(Pol)$. If the maximal sustainable level of pollution is $\bar{Pol} > 0$, then the emission constraint is $\pi \leq A(\bar{Pol})$. We thus set the level in equation (20) at $\bar{\pi} = \lambda A(\bar{Pol})$, $\lambda \leq 1$.

³ See R. M. Solow, "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, August 1957; E. Denison, *The Sources of Economic Growth*, Committee for Economic Development, 1962.

shows the reference path simulated by the simple model above and the simulations under the assumptions made above. Each simulation is shown with four population assumptions: Forrester's and the three alternative assumptions in Assumption A (thus B1 is assumption B and alternative population assumption A1).

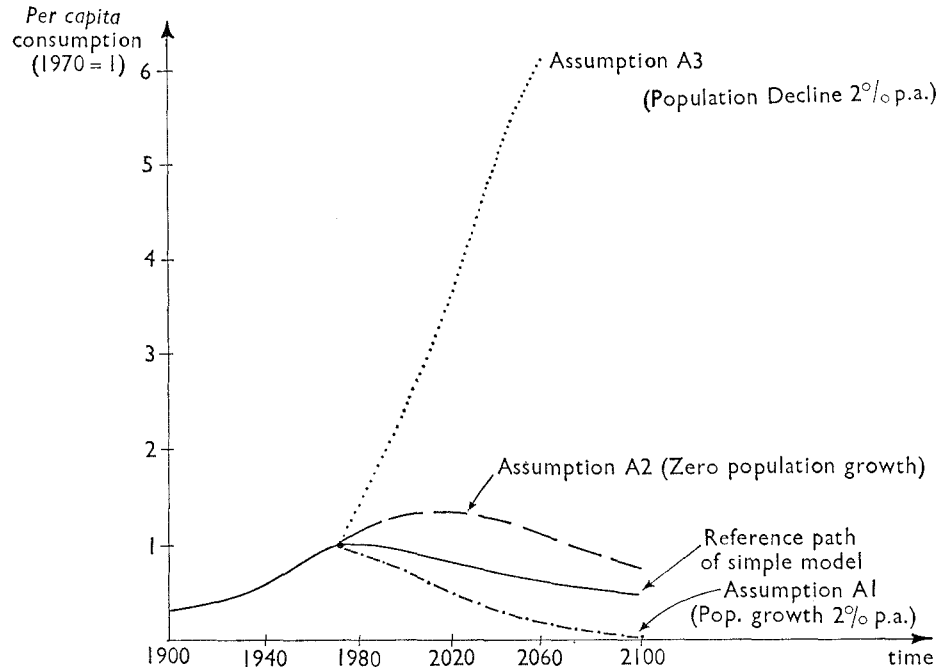


FIG. 9. Results for Consumption in Simple Model, Alternative Population Paths.

Figs. 9 to 12 show the path of *per capita* consumption along a reference path and along paths which vary some of the assumptions made by Forrester. The *reference path* is the path traced out by the simple model outlined in Section V above and shown in Fig. 7. The reference path is essentially the same as that described by Forrester's full model in *World Dynamics*, Tables 4-1 and 4-2.

The alternative paths in Fig. 9 show what happens to the path of *per capita* consumption if a change is made in the population projection alone. Assumption A1 shows the effect of constant exogenous growth of population at 2% *per annum*. Assumption A2 shows the effect of zero population growth. Assumption A3 shows the effect of population declining at 2% *per annum*.

Fig. 9 shows the effect of alternative population assumptions alone. It is clear from a casual examination of the model (in particular, considering the fixed stock of non-renewable resources) that some population *decline* is necessary to enable a continuing rise in consumption. Zero population growth (ZPG) is not sufficient. Fig. 9 indicates that population decline at 2% is sufficiently low to allow a continual rise in consumption rate. The assumptions about population are clearly crucial. After the basic results in the paper were derived, Mr. Warren Seering of Stanford University showed me simulations of the full Forrester model in which cases A1, A2, and A3

were run. He simply replaced Forrester's population dynamics with constant exogenous growth rates of 2.0, 0.0, and -2.0% annually. The paths for consumption were very similar to those shown in Fig. 9. The full model A1 path followed that in Fig. 9 almost exactly. The full model A2 path rose slightly higher then descended more steeply than the path in Fig. 9. The full model A3 path ran off the graph in 1994 with $c = 2$. It is thus confirmed that, in Forrester's *full* model, a policy leading to population decline is by itself sufficient to overcome all the obstacles to survival that Forrester's world view presents. This result directly contradicts Forrester's unsubstantiated contention that population control will fail (see p. 1159 above).

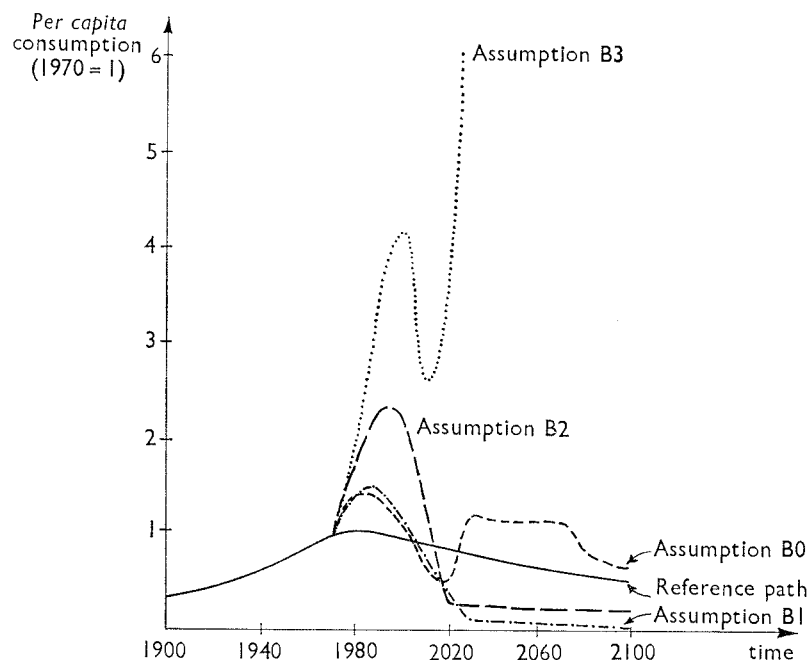


FIG. 10. Results for Simple Model with Proportional Savings Assumption and Alternative Population Paths.

For general description of assumptions, see Fig. 9. Fig. 10 shows the effect of changing population assumptions and savings assumptions. Assumption B changes the savings assumption to a constant proportion (rather than a declining fraction) of output. Assumption B0 uses Forrester's population assumption along with the change in savings assumption. Assumptions B1, B2, and B3 assume the constant savings rate and assume that population is growing at 2.0%, 0.0% and -2.0% *per annum*, respectively.

Fig. 10 shows the effect of the savings rate and indicates that this is by itself not a crucial assumption: Assumption B0 gives a line which keeps fairly close to the reference path (if judged by the enormous departures caused by adding a change in population assumptions). Fig. 11 shows the effects of input-reducing technological change. The effect of changing

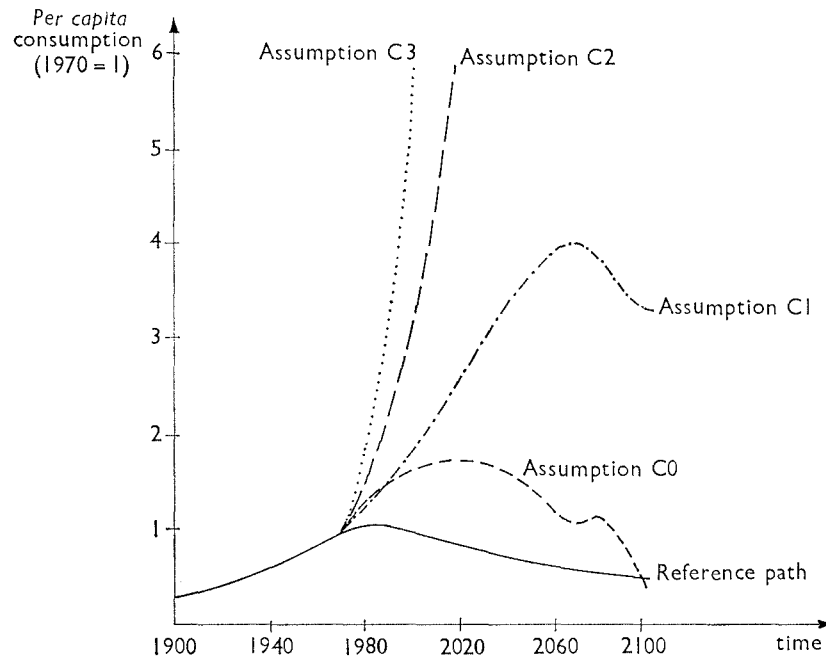


FIG. 11. Results for Simple Model with Technological Change and Alternative Population Paths.

For general description of assumptions, see Fig. 9. Fig. 11 assumes that there is input-reducing technological change at rate 2.5% per annum in all production relations. Assumption C0 uses Forrester population dynamics. Assumptions C1, C2, C3 assume input reducing technological change and further assume that population is growing at 2.0%, 0.0%, and -2.0% per annum, respectively.

this assumption is clearly to improve the prospects. Even with Forrester's population rules, the C0 line is well above the reference path until AD 2100, and in the case of population growth at 2% there is reasonably rapid growth of *per capita* consumption for about 100 years.

Finally in Fig. 12 overleaf we show the results of assuming a technology which allows substitution. This shows substantial growth for all population assumptions (including Forrester's assumed reproductive behaviour).

Although these simulations are perhaps a step toward greater empirical reality, they should not be construed as serious attempts to predict the future. Rather the purpose is to show that the model in *World Dynamics* has very different implications if alternative and more realistic assumptions are made on one or two key points,¹ e.g., about population growth, technological change, or the lack of substitutability. If these assumptions are modified in the direction of greater plausibility, the system behaves in a completely different fashion.

¹ There is an opinion abroad that Forrester's results are insensitive to the assumptions. Professor Dennis Gabor writes (letter to *Science*, April 14, 1972): "Forrester's world model [makes assumptions which] are validated [not] only by this plausibility, but more important by the *insensitivity* of the results to the details of the assumptions." Also see *World Dynamics*, p. 15.

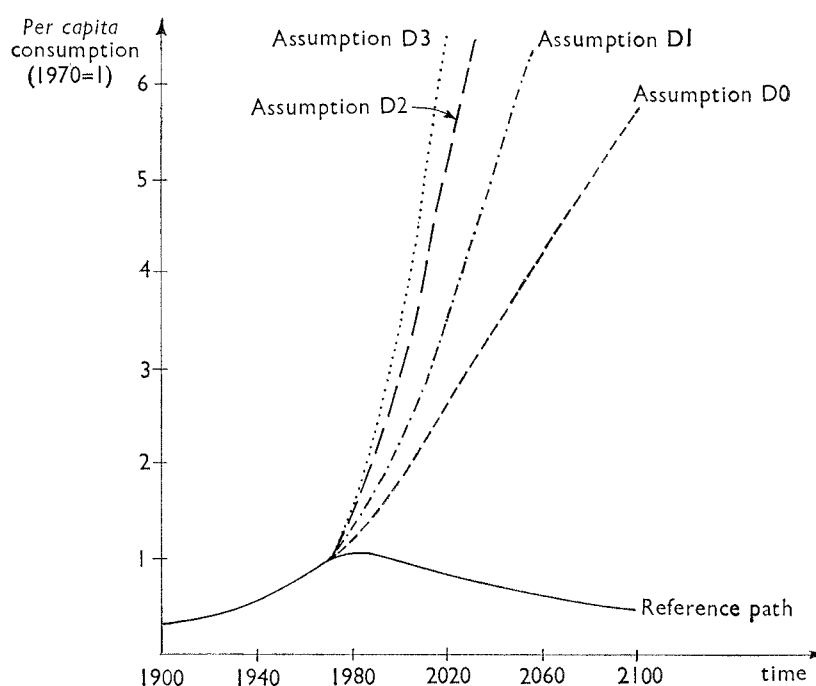


FIG. 12. Results for Simple Model with Substitution and Alternative Population Paths.

For a general description of assumptions, see Fig. 9. The simulations in Fig. 12 replace Forrester's production relations with a generalised log-linear production function described in equation (19). Assumption D0 uses Forrester's population dynamics. Assumptions D1, D2, and D3 use the new production function *and* further assume that population is growing at 2.0, 0.0, and -2.0% *per annum*, respectively.

VIII. CONCLUSIONS

What is the overall impression after a careful reading of *World Dynamics*?

First, the dynamic theory put forward in the work represents no advance over earlier work. The basic notions of system dynamics—usually called simultaneous difference or differential equations—have been used extensively in economics and elsewhere for decades.

Second, the economic theory put forth in *World Dynamics* is a major retrogression from current research in economic growth theory. *World Dynamics* contains no clear concepts of production functions, consumption, or output; nor is there any discernible method of allocating resources over time or between sectors.

Third, Forrester has made no effort in *World Dynamics* to identify any relation between his model and the real world. There is no explicit or apparent reference to data or existing empirical studies.

Fourth, the methodology of modelling in *World Dynamics* differs significantly from other studies of economic systems. *World Dynamics* constructs a

world model using assumptions which are intuitively plausible to the author, but without reference to current knowledge. The behaviour of this world model is then examined by calculating the dynamic path of the variables. Whereas most scientists would require empirical validation of either the assumptions or the predictions of the model before declaring its truth content, Forrester is apparently content with subjective plausibility. This discrepancy in scientific standards of acceptability is probably what lies behind the dispute about the value of *World Dynamics*.

Fifth, the predictions of the world's future are highly sensitive to the specification of the model. Simulations given above indicate that if assumptions regarding population, technological change, or substitution are changed, Forrester's model behaves in a dramatically different manner.

Sixth, there is some lack of humility toward predicting the future. Can we treat seriously Forrester's (or anybody's) predictions in economics and social science for the next 130 years? Long-run economic forecasts have generally fared quite poorly. Marx predicted the immiseration of the working class under capitalism; Keynes guessed¹ that capital could have no net productivity by the present year (1973); Galbraith assured us that scarcity is obsolete. And now, without the scantest reference to economic theory or empirical data, Forrester predicts that the world's material standard of living will peak in 1990 and then decline. *Sic transit gloria.*

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Date of receipt of final typescript: May 1973.

¹ See Keynes, Collected Works Vol. VII. *The General Theory*, p. 220.